18th Australasian Fluid Mechanics Conference
Launceston, Australia
3-7 December 2012

Onset of asymmetry and three-dimensionality in transitional round fountains in a linearly stratified fluid

Wenfeng Gao[1], **Wenxian Lin**[1,2], **Tao Liu**[1] **and S. W. Armfield**[3]

[1]Solar Energy Research Institute
Yunnan Normal University, Kunming, Yunnan 650092, China

[2]School of Engineering and Physical Sciences
James Cook University, Townsville, QLD 4781, Australia

[3]School of Aerospace, Mechanical and Mechatronic Engineering
The University of Sydney, NSW 2006, Australia

Abstract

Fountains injected into stratified fluids are widely found in nature and engineering settings. The onset of asymmetry, three-dimensionality and entrainment which occurs in transitional fountains is the key to shed light on the turbulence generation mechanism in fountains. It is also apparent that the stratification of the ambient fluid has significant effects on the onset of asymmetry, three-dimensionality and entrainment in fountains and a study on such effects will provide further insight into the interaction between stratification and turbulence generation mechanism in fountains. In this study, a series of direct numerical simulations were carried out for transitional round fountains in the ranges of $1 \leq Fr \leq 8$, $100 \leq Re \leq 500$ both in a homogeneous fluid and in a linearly stratified fluid with a constant dimensionless stratification $s = 0.03$, aiming at providing insights into the onset of asymmetry and three-dimensionality in transitional round fountains in a linearly stratified fluid when compared to that in a homogeneous fluid. The results show that a critical Re exists between 100 and 200 for $Fr = 2$ fountains which divides the fountains as either axisymmetric or asymmetric and three-dimensional; similarly a critical Fr exists between 1 and 2 for fountains at $Re = 200$ which divides the fountains as either axisymmetric or asymmetric and three-dimensional; when $Re \geq 200$, the $Fr = 2$ fountains maintain axisymmetry only in the developing stage and become asymmetric and three-dimensional at fully developed, steady state, and the stratified case has in general stronger extents of asymmetry and three-dimensionality than the homogeneous case; when $Fr \geq 2$, the fountains at $Re = 200$ maintain axisymmetry only in the developing stage and become asymmetric and three-dimensional at fully developed, steady state, and the stratified case has much stronger extents of asymmetry and three-dimensionality than the homogeneous case; The occurrence of asymmetry and three-dimensionality becomes much earlier when Re or Fr increases.

Introduction

Fountains are presented in many industrial and environmental settings, such as in natural ventilation, volcanic eruptions, cumulus clouds, reverse cycle air-conditioning, to name a few. A fountain occurs whenever a heavier fluid is injected vertically upward into a lighter fluid or a lighter fluid is injected vertically downward into a heavier fluid. In both cases buoyancy opposes the momentum of the jet flow, leading to gradually reduced vertical jet velocity until it becomes zero at a certain finite height. After that, the jet flow changes its direction and flows back around the core of the upward or downward flow and an intrusion forms on the bottom which moves outwards.

In a homogeneous ambient fluid, the fountain behavior is mainly governed by the Reynolds Number Re and the Froude Number Fr, which are defined as follows,

$$Re = \frac{W_0 X_0}{\nu}, \tag{1}$$

$$Fr = \frac{W_0}{\sqrt{gX_0(\rho_0-\rho_a)/\rho_0}} = \frac{W_0}{\sqrt{gX_0\beta(T_a-T_0)}}, \tag{2}$$

where X_0 is the radius of the fountain source, W_0 is the mean inlet velocity of the jet fluid at the source, g is the acceleration due to gravity, ρ_0, T_0 and ρ_a, T_a are the densities and temperatures of the jet fluid and the ambient fluid at the source, and ν and β are the kinematic viscosity and the coefficient of volumetric expansion of fluid, respectively. The second expression of Fr in equation (2) applied when the density difference is due to the difference in temperature of the jet and ambient fluids using the Oberbeck-Boussinesq approximation.

When the ambient fluid is linearly stratified, the fountain behavior will also be governed by the stratification parameter S_p,

$$S_p = -\frac{1}{\rho_{a,0}}\frac{d\rho_{a,Z}}{dZ}, \tag{3}$$

where $\rho_{a,0}$ and $\rho_{a,Z}$ are the densities of the ambient fluid at the bottom (i.e., at $Z = 0$) and at height Z. If the Oberbeck-Boussinesq approximation is valid, S_p can also be represented by the temperature stratification parameter S,

$$S = \frac{dT_{a,Z}}{dZ} = \frac{S_p}{\beta}, \tag{4}$$

where $T_{a,Z}$ is the temperature of the ambient fluid at the height Z. However, the dimensionless form of S, as defined below, is normally used instead,

$$s = \frac{d\theta_{a,z}}{dz} = \frac{X_0}{(T_{a,0}-T_0)}S = \frac{X_0}{\beta(T_{a,0}-T_0)}S_p, \tag{5}$$

where $\theta_{a,z} = (T_{z,Z}-T_{a,0})/(T_{a,0}-T_0)$ and $z = Z/X_0$ are the dimensionless temperature of the ambient fluid at height Z and the dimensionless height, respectively.

A fountain can be classified as either “very weak” ($0 < Fr < 1$), “weak” ($1 < Fr < 3$), or “forced” ($Fr > 3$) [2]. The major features of a forced fountain include the buoyancy force is weaker than the source momentum flux (thus also called “strong” fountains); the inner upflow of the fountain fluid behaves like a turbulent jet with strong mixing and entrainment of the ambient fluid (thus also called “turbulent” fountains) while the outer downflow of the fountain fluid behaves more like a dense plume; both the upflow and downflow continue to develop along their trajectories so the flow never attains self-similarity and the flow statistics vary with the axial location and the Froude number; and the fountain penetrates a large distance into the ambient fluid. On the other hand, in very weak or weak fountains, the discharge momentum flux plays a less important role than the negative buoyancy flux and the flow is in the laminar or transitional regime (thus also called "laminar" or "transitional" fountains). For these weak fountains, it has been shown that their flow behavior is considerably different from that of forced fountains (see, e.g., [1, 2, 3, 6]). For example, it has been shown that Z_m is smaller than R_0 for weak fountains while for forced fountains Z_m is much larger than R_0, where Z_m is the maximum fountain penetration height; there are no distinguishable upward and downward flows in weak fountains, instead, the streamlines curve and spread from the fountain sources, while in forced fountains, the upward and downward flows are clearly distinguished; there is usually little entrainment of the ambient fluid into the fountain fluid in weak fountains while such an entrainment is one of the major activities occurring in forced fountains; the Reynolds number affects the penetration height in laminar fountains whereas in forced fountains it does not.

The onset of instability and unsteadiness in fountains is the key to elucidate the mechanism for the generation of turbulence and entrainment in fountains but is not well understood, although some attempts have been made recently for transitional fountains in homogeneous fluids [5, 6, 7]. Nevertheless, no study has been found to explore the onset of instability and unsteadiness in transitional fountains in stratified fluids where the stratification of the ambient fluid complicates the mechanism of the generation of turbulence and entrainment in fountains, which motivates this study.

In this study, a series of direct numerical simulations (DNS) were carried out for transitional round fountains in the ranges of $1 \leq Fr \leq 8$, $100 \leq Re \leq 500$ both in a homogeneous fluid and in a linearly stratified fluid with a constant dimensionless stratification $s = 0.03$, aiming at providing insights into the onset of asymmetry and three-dimensionality in transitional round fountains in a linearly stratified fluid when compared to that in a homogeneous fluid.

Numerical Methods

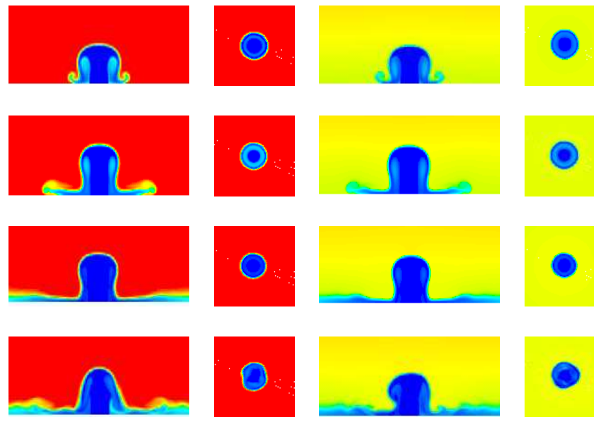
The physical system under consideration is a vertical circular container containing a Newtonian fluid initially at rest and at either a uniform temperature of T_a (in the homogeneous case) or a constant temperature gradient $dT_{a,Z}/dZ$ (in the stratified case), the sidewall is non-slip and insulated and the top is open. On the bottom center, an orifice with radius X_0 is used as the fountain discharge source. The remaining bottom region is a rigid non-slip and insulated boundary. At time $t = 0$, a stream of fluid at T_0 ($T_0 < T_a$ for the homogeneous case or $T_0 < T_{a,0}$ for the stratified case) is injected upward into the container from the source to initiate the fountain flow and this discharge is maintained thereafter.

The flow is governed by the Navier-Stokes and temperature equations, which are discretized on a non-uniform mesh using finite volumes, with standard 2nd-order central difference schemes used for the viscous and divergence terms. The 3rd-order QUICK scheme is used for the advective terms. The 2nd-order Adams-Bashforth and Crank-Nicolson schemes are used for the time integration of the advective terms and the diffusive terms. The PRESTO (PREssure STaggering Option) scheme is used for the pressure gradient. The ICEM technique is used to create O-Type Multiblock Hexahedron meshes. The numbers of grids used are in the range of 4 to 5 millions. All DNS were carried out using Ansys Fluent 13.

Results and Discussions

Typical Evolution of Fountain Flows

The typical evolution of fountain flows, both in homogeneous and in linearly stratified fluids, is presented in figure 1 for $Fr = 2$ and $Re = 200$. The dimensionless temperature stratification for the stratified case is $s = 0.03$. The figure clearly shows that although this $Fr = 2$ and $Re = 200$ fountain, both in the homogeneous and stratified cases, maintains axisymmetry for a very long time since initiation, it eventually becomes asymmetric and three-dimensional at a quite late stage of development.



Homogeneous case   Stratified case

Figure 1: Typical evolution of a $Fr = 2$ fountain at $Re = 200$ in both a homogeneous fluid (two columns on the left) and a linearly stratified fluid at $s = 0.03$ (two columns on the right). The first and third columns are temperature contours in the vertical plane through the symmetry axis of the cylinder, and the second and the fourth columns are temperature contours in the horizontal plane at height $Z = 0.5Z_{m,i}$, where $Z_{m,i}$ is the initial fountain penetration height. The gravity acts in the negative vertical direction. The top to the bottom rows correspond to times $t = 20$, 40, 120 and 200, respectively, where the time t is made dimensionless by X_0/V_0.

Behavior under the Effect of *Re*

Figure 2 presents the temperature contours of $Fr = 2$ fountains with $Re = 100$, 200, 300, 400 and 500 in both a homogeneous fluid and a linearly stratified fluid at steady state. It shows that the $Re = 100$ fountain maintains axisymmetry all the times, for both cases. However, when $Re \geq 200$, the $Fr = 2$ fountains become asymmetric and three-dimensional for both cases. It is clear that a critical Re exists between 100 and 200 for $Fr = 2$ fountains which divides the fountains to become asymmetric and three-dimensional or not. However, to determine this critical Re value, further simulations must be carried out which is beyond of the scope of the current study. It is also speculated that for each of the other Fr fountains there should be a critical Re to divide the fountains as axisymmetric or asymmetric and three-dmensional.

A more clear indication of the onset of asymmetry and three-dimensionality in a round fountain is represented by the tangent velocity on the interfacial surface between the fountain fluid and the ambient fluid, which is defined as the surface where the fluid temperature is $[T_a - 0.01(T_a - T_0)]$ for the homogeneous case or $[T_{a,Z} - 0.01(T_{a,Z} - T_0)]$ for the stratified case. For a round fountain that maintains axisymmetry the tangent velocity on the interfacial surface between the fountain fluid and the ambient fluid must be zero (theoretically) or negligibly small (due to inevitable numerical errors in DNS). Such information is presented in figure 3 for the $Fr = 2$ fountains with varying Re in both a homogeneous fluid and in a linearly stratified fluid, where the time series of V_t/V_0 of each fountain are shown. The results clearly show that for these $Fr = 2$ fountains, when $Re = 100$, V_t/V_0 is less than 0.1% all the times, indicating this fountain maintains axisymmetry anytime, which is in agreement with the results presented in figure 2. Nevertheless, when $Re \geq 200$, the $Fr = 2$ fountains maintain axisymmetry only in the developing stage (with $V_t/V_0 \approx 0$) and become asymmetric and three-dimensional at fully developed, steady state, with the tangent velocity at the order of the fountain inlet velocity. It is also observed that for the same Re, in general the $Fr = 2$ fountain in the stratified case has stronger extents of asymmetry and three-dimensionality than that in the homogeneous case and the occurrence of such asymmetry and three-dimensionality is also earlier than the homogeneous case. In a linearly stratified fluid, the fountain penetrates a shorter height, due to the restriction of the stratification, than in a homogeneous fluid and hence the fountain flow falls down earlier in the stratified case, which in turn leads to earlier occurrence of the asymmetry and three-dimensionality. Furthermore, it is clear that the occurrence of asymmetry and three-dimensionality in the $Fr = 2$ fountains becomes, in general, earlier when Re increases.

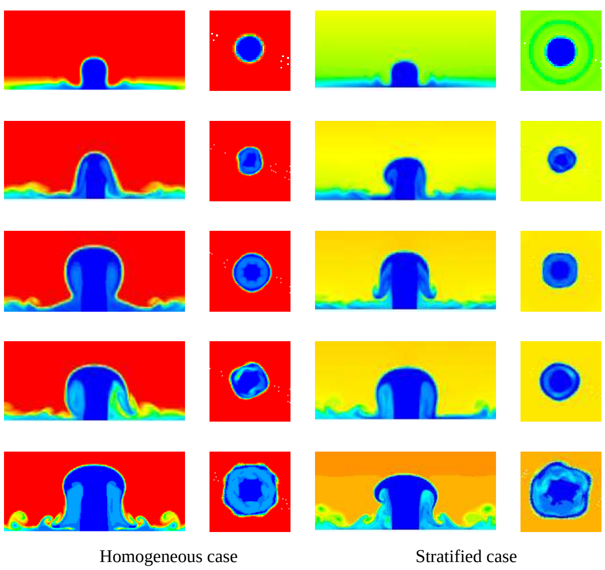


Figure 2: Steady-state temperature contours of $Fr = 2$ fountains with varying Re in both a homogeneous fluid and a linearly stratified fluid at $s = 0.03$. The first and third columns are temperature contours in the vertical plane through the symmetry axis of the cylinder, and the second and the fourth columns are temperature contours in the horizontal plane at height $Z = 0.5Z_{m,i}$. The top to the bottom rows correspond to $Re = 100$, 200, 300, 400 and 500, respectively.

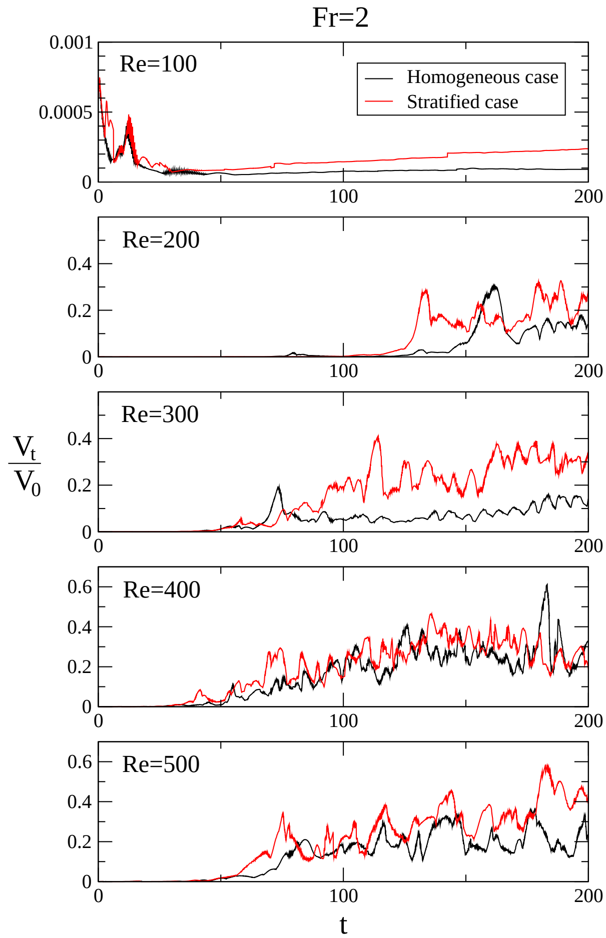


Figure 3: Time series of V_t/V_0 of $Fr = 2$ fountains with varying Re in both a homogeneous fluid and a linearly stratified fluid at $s = 0.03$, where V_t is the tangent velocity on the interfacial surface between the fountain fluid and the ambient fluid. The time t is made dimensionless by X_0/V_0.

Behavior under the Effect of Fr

Figure 4 presents the temperature contours of $Fr = 1$, 2, 3, 5 and 5 fountains, all at $Re = 200$, in both a homogeneous fluid and a linearly stratified fluid at steady state. It clearly shows that the $Fr = 1$ fountain maintains axisymmetry all the times, for both cases. However, all $Fr \geq 2$ fountains become asymmetric and three-dimensional for both cases. It is clear that a critical Fr exists for $Re = 200$ which divides the fountains to become asymmetric and three-dimensional or not. However, similar to the critical Re case as addressed above, to determine this critical Fr value, further simulations must be carried out which is again beyond of the scope of the current study. It is also speculated that for each of the other Re values there should be a critical Fr which divides the fountains as axisymmetric or asymmetric and three-dmensional.

The time series of V_t/V_0 of $Fr = 1$, 2, 3, 5 and 5 fountains, all at $Re = 200$, are shown in figure 5 for both the homogeneous and stratified cases. The results clearly show that the $Fr = 1$ fountains maintains axisymmetry all the times, as V_t/V_0 is less than 0.1% anytime, which is in agreement with the results presented in figure 4. Nevertheless, when $Fr \geq 2$, these fountains at $Re = 200$ maintain axisymmetry only in the developing stage (with $V_t/V_0 \approx 0$) and become asymmetric and three-dimensional at fully developed, steady state, with the tangent velocity at the order of the fountain inlet velocity. It is also observed that for the same Fr, in general the $Re = 200$ fountain in the stratified case has much stronger extents of asymmetry and three-dimensionality than that in the homogeneous case and the occurrence of such asymmetry and three-dimensionality is also earlier than the homogeneous case, with the same reasoning as discussed above. Furthermore, it is observed that the occurrence of asymmetry and three-dimensionality in the $Re = 200$ fountains becomes much earlier when Re increases. In the $Fr = 8$ case, the fountain in the stratified case becomes asymmetric and three-dimensional at about $t = 30$, while that in the homogeneous case, this happens at about $t = 90$.

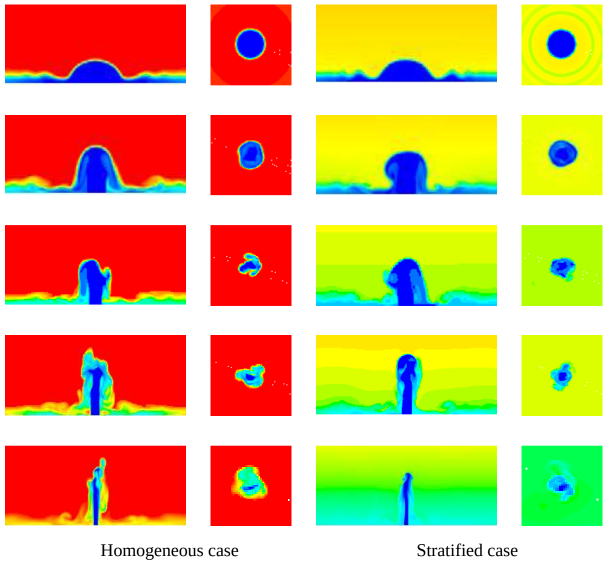


Figure 4: Steady-state temperature contours of fountains at $Re = 200$ with varying Fr in both a homogeneous fluid and a linearly stratified fluid at $s = 0.03$. The first and third columns are temperature contours in the vertical plane through the symmetry axis of the cylinder, and the second and the fourth columns are temperature contours in the horizontal plane at height $Z = 0.5Z_{m,i}$. The top to the bottom rows correspond to $Fr = 1$, 2, 3, 5 and 8, respectively.

Conclusions

The direct numerical simulations for transitional round fountains in the ranges of $1 \le Fr \le 8$, $100 \le Re \le 500$ both in a homogeneous fluid and in a linearly stratified fluid with a constant dimensionless stratification $s = 0.03$ show that the onset of asymmetry and three-dimensionality in a transitional fountain can be detected by the tangent velocity on the interfacial surface between the fountain fluid and the ambient fluid. The results also demonstrate that a critical Re exists between 100 and 200 for $Fr = 2$ fountains, or a critical Fr exists between 1 and 2 for fountains at $Re = 200$, which divides the fountains as either axisymmetric or asymmetric and three-dimensional. When $Re \ge 200$, the $Fr = 2$ fountains maintain axisymmetry only in the developing stage and become asymmetric and three-dimensional at steady state, and the stratified case has in general stronger extents of asymmetry and three-dimensionality than the homogeneous case. When $Fr \ge 2$, the fountains at $Re = 200$ maintain axisymmetry also only in the developing stage and become asymmetric and three-dimensional at steady state, and the stratified case has much stronger extents of asymmetry and three-dimensionality than the homogeneous case. For all these fountains the occurrence of asymmetry and three-dimensionality in the stratified case is earlier than in the homogeneous case, and the occurrence of asymmetry and three-dimensionality becomes much earlier when Re or Fr increases. It is apparent that the determination of the exact critical values of Re or Fr for transitional fountains needs further studies.

Acknowledgements

The support from the National Natural Science Foundation of China (11072211, 50879074), the Yunnan Natural Science Foundation (2011FA017), the Program for Changjiang Scholars and Innovative Research Team in University of Ministry of Education of China, and the Australian Research Council is gratefully acknowledged.

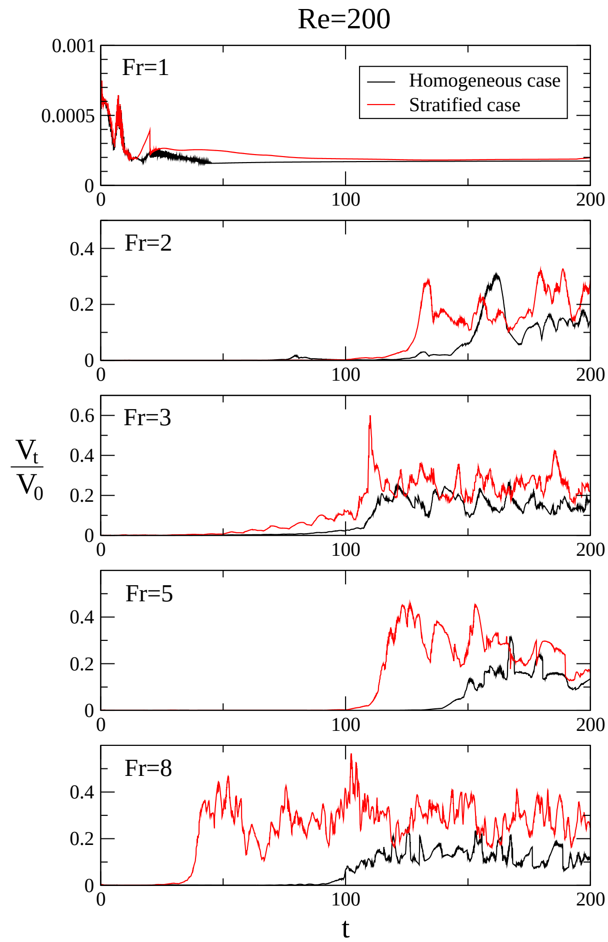


Figure 5: Time series of V_t/V_0 of $Re = 200$ fountains with varying Fr in both a homogeneous fluid and a linearly stratified fluid at $s = 0.03$.

References

[1] Burridge, H.C. and Hunt, G.R., The Rise Heights of Low- and High-Froude-Number Turbulent Axisymmetric Fountains, *J. Fluid Mech.*, **691**, 2012, 392–416.

[2] Kaye, N.B. and Hunt, G.R., Weak Fountains, *J. Fluid Mech.*, **558**, 2006, 319–328.

[3] Lin, W. and Armfield, S.W., Direct Simulation of Weak Axisymmetric Fountains in a Homogeneous Fluid, *J. Fluid Mech.*, **403**, 2000, 67–88.

[4] Lin, W. and Armfield, S.W., The Reynolds and Prandtl Number Dependence of Weak Fountains, *Comput. Mech.*, **31**, 2003, 379–389.

[5] Lin, W. and Armfield, S.W., Onset of Entrainment in Transitional Round Fountains, *Int. J. Heat Mass Transfer*, **51**, 2008, 5226–5237.

[6] Williamson, N., Srinarayana, N., Armfield, S.W., McBain, G. and Lin, W., Low Reynolds Number Fountain Behavior, *J. Fluid Mech.*, **608**, 2008, 297–317.

[7] Williamson, N., Armfield, S.W. and Lin, W., Transition Behavior of Weak Turbulent Fountains, *J. Fluid Mech.*, **655**, 2010, 306–326.